Corrigenda to 'A shorter model theory', Wilfrid Hodges

My thanks to people who sent me corrections, and in particular to Dan Auerbach, Peter Cameron, Fredrik Engstrom, Sol Feferman, Steffen Lempp, Javier Moreno, Soren Riis, Malcolm Schonfield, Paul Tveite and Markus Vasquez. My apologies that this list of corrigenda may not be up to date; I am trying to find time to remedy that.

p. viii Last line, the ftp site at Queen Mary no longer seems to be operational. Instead the URL should be `http://wilfridhodges.co.uk/corrigmt.pdf`.

p. 5 Exercise 1 Experience with classes shows that I should have defined 'asymmetric'. A binary relation R is *asymmetric* if there are no a and b such that both Rab and Rba.

p. 15 Exercise 4 For 'generate A' read 'lists the elements of A'.

p. 36 formula (2.33) There should be $\bigwedge_{n<\omega}$ at the beginning of the formula.

p. 66 Exercise 2 The elimination set needs to include equations too.

p. 76 end of proof The final B should be C.

p. 84 l. 12 $\Theta_{n,r}$ should be $\Theta_{n,k}$.

p. 84 formula (3.7) The second $\bigwedge$ should be a $\bigvee$.

p. 85 l. 7 D should be B.

p. 121 ll. 3–18 These should read:

> 'is finite and non-empty. We prove the claim by induction on i. When $i = 0$ it is trivially true. Assuming it is true for i, we prove it for $i + 1$ as follows.
>
> Let $(a_0, \ldots, a_{i-1})$ be some tuple in U_i, and consider the non-empty set W of all elements a of A such that
>
> $$A \models \exists x_{i+1} \ldots x_{n-1} \theta(a_0, \ldots, a_{i-1}, a, x_{i+1}, \ldots, x_{n-1}, \bar{b}).$$
>
> Since W is a definable subset of A and A is a minimal structure, there are two possibilities: either W is finite, or $(\text{dom } A) \setminus W$ is finite. In the first case suppose W has exactly k elements. Then take ψ_{i+1} to be the formula
>
> $$\psi_i \ \wedge \ \exists_{=k} x_i \exists x_{i+1} \ldots x_{n-1} \theta(\bar{x}, \bar{y}) \ \wedge \ \exists x_{i+1} \ldots x_{n-1} \theta(\bar{x}, \bar{y}).$$
>
> In the second case, since the field of algebraic elements of A is infinite, W must meet it, say in some element a. Then a satisfies some nontrivial polynomial equation, say $p(x) = 0$. We take ψ_{i+1} to be the formula
>
> $$\psi_i \ \wedge \ (p(x_i) = 0) \ \wedge \ \exists x_{i+1} \ldots x_{n-1} \theta(\bar{x}, \bar{y}).$$

This proves the claim.

When $i = n$, the claim gives us a formula $\psi(\bar{x}, \bar{y})$ (namely $\psi_n(\bar{x}, \bar{y}) \wedge \theta(\bar{x}, \bar{y})$) such that just finitely many tuples $\bar{a}$ (viz. those in U_n) satisfy $\psi(\bar{x}, \bar{b})$ in A, and they all lie in $\bar{b}/\theta$. We need a trick to turn this finite set U_n into a single tuple; the tuple must be determined by the set, independent of any ordering of the set. Here we use algebra. For each tuple $\bar{a} = (a_0, \ldots, a_{n-1})$ in U_n we take $q_{\bar{a}}(X_0, \ldots, X_n)$ to be the polynomial

$$a_0 X_0 + \ldots + a_{n-1} X_{n-1} + X_n.$$

Then we write $q(X_0, \ldots, X_n)$ for the polynomial

$$\prod_{\bar{a} \in U_n} q_{\bar{a}}.$$

By the unique factorisation theorem for polynomial rings over a field, the set of irreducible polynomials $q_{\bar{a}}$, and hence the set U_n, can be recovered from q. So for our tuple representing $\bar{b}/\theta$ we can take the sequence of coefficients of q, for some fixed ordering of the monomials. □

p. 123 Last paragraph This should read:

'A good route into Ehud Hrushovski's model-theoretic proof of the function field case of the Mordell-Lang conjecture (from diophantine geometry) is the following book:

> Bouscaren, E. (ed.). Model Theory and Algebraic Geometry, An introduction to E. Hrushovski's proof of the geometric Mordell-Lang-conjecture. Lecture Notes in Mathematics 1696. Berlin, Springer-Verlag, 1998.

One of the central lemmas used in Hrushovski's argument is well worth studying for its own sake; it gives a model-theoretic axiomatisation of the Zariski topology: ' (etc. as before).

p. 129 Exercise 9 For '$\phi(B)$' read '$|\phi(B)|$'.

p. 137 (3.5) for 'of L' read 'of L, $\bar{c}$ in C'.

p. 138 l. 1f For 'C' read 'C''.

p. 145f Transpose the diagrams on these two pages.

p. 153 l. 4 For '$2^{|L|}$' read '$2^{|\Phi|}$'.

p. 154 l. -7 For '$[Y]^{k-1}$' read '$[Y]^{k}$'.

p. 179 l. 8 Delete '$(\bar{a})$' twice.

p. 189 l. 4 For 'A_δ' read 'A_λ'.

p. 200 Exercise 4 For 'by Lindström's test' read 'by adapting Lindström's test', and at end add '[Consider saturated countable models.]'.

p. 208 Exercise 14 In the middle of the exercise, $\bigwedge$ should be $\bigvee$.

p. 209 ll. 7–9 The Wilkie reference should be:

> 'Wilkie, A. Model completeness results for expansions of the ordered field of real numbers by restricted Pfaffian functions and the exponential function. Journal of the American Mathematical Society 9 (1996) 1051–1094.'

p. 212 l. 3 For '0-big' read '1-big'.

p. 241 l. -5 For '$\prod_I A_i$' read '$\prod_I A_i/\mathcal{F}$'.

p. 242 l. 14 The third 'I' should be 'W'.

p. 252 (1.4) The second arrow should point to the left.

p. 279 ll. 4,7 Transpose b_0 and b_1.

p. 293 l. -1 For 'Theorem 6.4.3' read 'Theorem 5.3.3'.

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