Ibn Sīnā's propositional logic

Wilfrid Hodges
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http://wilfridhodges.co.uk/arabic43.pdf

We survey some main features of Ibn Sīnā's propositional logic. Supporting evidence is being written up (but it will be very tedious, involving close examination of many texts).

Ibn Sīnā's logic splits into two parts: predicative and propositional.
But the two parts are not completely separate.
Both are developments of the two-dimensional sentences that he introduces early in *Qiyās* and in *Easterners*.

Compare:

1. Every writer sometimes makes a mistake while writing.
2. There is a time when everybody writing at that time is making a mistake.
3. There is a time when everybody is writing and everybody is making a mistake.

We move from 1 to 3 by pushing the Aristotelian quantifier inwards and the time quantification outwards.
1 and 2 are forms of 2D sentences like those early in *Qiyās*.
3 is a typical propositional logic sentence.

So by rearranging pieces of 2D sentences, we reach forms

(a) Every time when p is a time when q.
(e) No time when p is a time when q.
(i) Some time when p is a time when q.
(o) Not every time when p is a time when q.

The analogy with (a), (e), (i), (o) assertoric sentences is obvious.

Ibn Sīnā develops this analogy in *Qiyās* vi.1 by describing the valid syllogisms for sentences of these forms.
His account is an *exact* parallel of his account of assertoric syllogisms in *Qiyās* ii.4.
It's also very close to Aristotle *Prior Analytics* i.4–6.

The listings agree not just in the syllogisms found valid, but also in the proofs of validity.

In fact the main difference seems to be that in both *Qiyās* ii.4 and vi.1 Ibn Sīnā gives an ecthetic proof of *Baroco*, not in Aristotle.

Since these proofs include use of (a) conversion,
Ibn Sīnā must be reading (a) with an extra clause:

'Every time when p is a time when q,
and there is a time when p.'

We call the added clause the *existential augment.*
Ibn Sīnā says that in the assertoric case almost all his predecessors assumed the augment.

The (a) propositional form above is close to a form already discussed by Aristotelians including Al-Fārābī.
Al-Fārābī calls it *muttaṣil* ('connective' in Shehaby).

Ibn Sīnā extends the name *muttaṣil* to the (e), (i) and (o) forms.
There is no solid evidence that anybody before Ibn Sīnā made this extension.
It seems to come from his two-dimensional analysis mentioned above.

Earlier Aristotelians also had another class of propositional compound sentences, which Al-Fārābī called *munfaṣil* ('separative' in Shehaby).

These had the form 'Either p or q'.
In the *strict* version this meant 'Exactly one of p and q is true';
in the *non-strict* version, 'At least one of p and q is true'.

Ibn Sīnā expands to 'At every time, either p or q is true', which he treats as universally quantified affirmative, i.e. an (a) sentence.

Ibn Sīnā notes that we have an equivalence

'At every time, either p or q is true.'

$$\Leftrightarrow$$

'Every time when not-p is a time when q.'

provided that we use the non-strict reading in the munfaṣil and drop the existential augment in the muttaṣil.

He uses this equivalence as the basis of an extension of the *munfaṣil* sentences to (e), (i) and (o) forms, each equivalent to the corresponding *muttaṣil* form by adding 'not' in appropriate places.
These equivalences assume no existential augments and no strictness.

We will use the notation (a,mt)(p,q) for the *muttaṣil* proposition with first clause p and second clause q.
Similarly (e,mt)(p,q), (i,mt)(p,q), (o,mt)(p,q).

Likewise for the *munfaṣil* sentences we write (a,mn)(p,q), (e,mn)(p,q), (i,mn)(p,q), (o,mn)(p,q).

We will come back later to English readings for these sentences.

In *Qiyās* vi.2 Ibn Sīnā develops a detailed theory of syllogisms that mix *muttaṣil* and *munfaṣil* sentences.
It contains dozens of implications between the forms, plenty enough to allow us to read off the following equivalences:

(a,mt)(p,q) $\Leftrightarrow$ (e,mt)(p,not q) $\Leftrightarrow$ (a,mn)(not p,q) $\Leftrightarrow$ (e,mn)(p,q).

(I think a modern logician would have tried

$$\ldots \Leftrightarrow \text{(e,mn)(not } p\text{,not } q\text{).}$$

But Ibn Sīnā was less sensitive than we are to issues of symmetry and completeness.)

Similarly for the (i) and (o) forms, which give contradictory negations of the (e) and (a) forms:

(o,mt)(p,q) $\Leftrightarrow$ (i,mt)(p,not q) $\Leftrightarrow$ (o,mn)(not p,q) $\Leftrightarrow$ (i,mn)(p,q).

Rescher 1963 gave first-order formalisations of *muttaṣil* and *munfaṣil* sentences.
Lacking *Qiyās*, he had to guess several of the formalisations.
In fact those for *muttaṣil* sentences are basically correct, but the equivalences above are incompatible with his guesses for (e,mn) and (i,mn) sentences.

Parenthetic remark:

The manuscripts contain quite a lot of readings at variance with the equivalences above, but they are a clear minority and they don't add up to any alternative system. A small cluster of passages support a rival system for the existential formulas:

(o,mt)(p,q) ⇔ (i,mt)(p,not q) ⇔ (o,mn)(p,not q) ⇔ (i,mn)(not p,not q)

But this doesn't build up to a plausible system overall.
In particular it's incompatible with reading (o,mn) and (i,mn) as contradictory negations of (a,mn) and (e,mn).
It seems that copyists and glossators, and perhaps Ibn Sīnā himself, found the equivalences confusing.

The equivalences above involve *metathetic negation*, i.e. negation of clauses as opposed to whole sentences.

Ibn Sīnā uses metathetic negation freely in his account of *muttaṣil-munfaṣil* syllogisms.
So we can speak of them as *metathetic syllogisms*.

Metathetic negation allows reversible conversions for all the *muttaṣil* and *munfaṣil* sentence forms, for example

(a,mt)(p,q) ⇒ (a,mt)(not q,not p).

Logical fact: There is just one kind of minimal inconsistent set of three sentences of these kinds, namely

(a,mn){not r,q}, (a,mn){not q,p}, (i,mt){not p,r}

where {,} indicates that the order doesn't matter.

All valid moods are got from this set of three sentences by fixing the order of clauses, taking two sentences as premises and the contradictory negation of the third as conclusion, possibly replacing letters s by 'not s', and using the equivalences above.

This yields four figures (Ibn Sīnā uses only the first three), and in each figure three moods according as the premises are

universal, universal
existential, universal
universal, existential.

There are three new non-Aristotelian moods, for example a first figure mood with second (= major) premise existential:

(a,mt)(r,q), (i,mt)(not q,p). Therefore (i,mt)(not r,p).

Ibn Sīnā develops a proof theory:

Step One: Translate both premises to *muttaṣil* forms.

Step Two: Deduce a *muttaṣil* conclusion if there is a suitable Aristotelian mood.

Step Three: If there is no suitable Aristotelian mood, use reversible conversions and perhaps permutation of premises to change the figure to one where a suitable Aristotelian mood is available. If necessary, use a reversible conversion on the conclusion.

Step Four: If wanted, translate the conclusion to another form, e.g. *munfaṣil*.

Ibn Sīnā is not interested in minimising his methods.
But the proof theory above will work if he always translates to (a,mt) or (i,mt) forms in Step One, and uses just three Aristotelian moods (best *Barbara*, *Darii* and *Disamis*) in Steps Two and Three.

Suhrawardi in the next century made a very similar reduction of predicative syllogisms.
By using metathetic negation he restricted to just affirmative sentence forms. He relied on just three moods (though not quite the same that would work for Ibn Sīnā above).

It's hard to reject the idea that Suhrawardi had Ibn Sīnā's *muttaṣil-munfaṣil* syllogistic at the back of his mind.

Besides listing and proving valid syllogisms, Ibn Sīnā also aims to show that certain formal premise-pairs are not productive in a given figure.
He follows Aristotle's method for assertoric premise-pairs.

In this method we give two interpretations of the premise-pairs, with the properties:

- In both interpretations the premise-pairs are true.
- If C and A are the subject and predicate terms for the figure, then 'Every C is an A' is true in the first interpretation and 'No C is an A' is true in the second.

Rationale: The premise-pair can't entail either 'No C is an A' or 'Some C is not an A', because 'Every C is an A', true in the first interpretation, contradicts both these.
Similarly it can't entail 'Every C is an A' or 'Some C is an A' because of the second interpretation.

Important point: 'Every C is an A' contradicts 'No C is an A' *only if the existential augment is assumed.*

Ibn Sīnā thinks he can ignore the existential augment, because he drops it in his metathetic syllogisms (and perhaps for other reasons). This is false.

As a result Ibn Sīnā claims to give proofs of unproductiveness for several premise-pairs that are in fact productive.

There is a simple test of productivity: a premise-pair is productive if and only if one of the occurrences of the middle clause is positive (= undistributed) and the other is negative (= distributed).
Ibn Sīnā could hardly have made these mistakes if he had been aware of this test.

This confirms what was likely from other evidence, that Ibn Sīnā had no notion of positive or negative occurrences.

Augments and additions

Ibn Sīnā mentions several features that can be added to (a,mt) or (a,mn) sentences.
All these features come from the earlier Aristotelian tradition. So any extension of them to (e), (i) or (o) sentences is probably Ibn Sīnā's own, and in Ibn Sīnā himself these extensions are very limited.

The features are:

- For (a,mn) sentences, strictness.
- For (a,mt) sentences, existential augment, being *ittifāqī*, being *luzūmī*.

In the formal theory of *Qiyās* vi.2, Ibn Sīnā considers **strictness** as an optional feature that can be added to (a,mn) sentences.
He develops versions of his logic with it and without it.

In practice he starts to do something similar with the **existential augment**, though much more erratically.
In *Qiyās* vii.1 and vii.2 and viii.3 the existential augment vanishes altogether.

The ***ittifāqī*** and ***luzūmī*** classifications are unclear. What follows is partly guesswork.

In some places (mainly where he is studying earlier Aristotelian notions) Ibn Sīnā suggests that every *muttaṣil* sentence is either *ittifāqī* or *luzūmī*.
Elsewhere he suggests that none are *ittifāqī* or *luzūmī* in their 'absolute' form, but some (in practice only (a,mt) and (e,mt)) can have one of these features added.

The 'absolute' forms are presumably those we have been studying above.

It seems that the notions are strictly not logical at all, though Ibn Sīnā tries to give them logical content.
They come from Peripatetic speculations about how we can know that a sentence ‘If p then q’ is true.

Two suggestions were:
(a) We can know it because we know that q is true.
(b) We can know it because we can deduce q from p.

Ibn Sīnā reads the *ittifāqī* case as (a)
and the *luzūmī* case as (b).

ittifāqī probably translates Greek *kata sumbebēkos*, which goes into Latin as *secundum accidens.* The Arabic could mean either ‘by chance’ or ‘to do with agreement’.

Shehaby opts for ‘chance’, but there is no element of chance in most of the examples Ibn Sīnā gives for the notion.

The main common feature is that we know ‘Whenever p then q’ because we know ‘Always q’.
So a better reading is that an *ittifāqī* (a,mt) statement is known to be true because we know that its second clause *agrees with* the way the world is (the *wujūd*, as Ibn Sīnā says).

This is confirmed by Ibn Sīnā’s extension of the notion to (e,mt) sentences, which he says ‘deny *ittifāq*’.
The natural reading is that these (e,mt) sentences are known to be true because their second clause is known to *disagree with* the way the world is.

Ibn Sīnā adds that if ‘Whenever p then q’ is *ittifāqī* and we combine it with ‘p’ (or maybe ‘Always p’) to deduce ‘q’ (or maybe ‘Always q’), then the inference gives no new information.
Formally his point is a natural deduction reduction rule.
Compare Prawitz on $\supset$-reduction, *Natural Deduction* p. 37:

$$\dfrac{\begin{matrix}\vdots\\ A\end{matrix} \qquad \dfrac{\begin{matrix}\vdots\\ B\end{matrix}}{(A \supset B)}}{\begin{matrix}(B)\\ \vdots\end{matrix}} \qquad\qquad \begin{matrix}\vdots\\ (B)\\ \vdots\end{matrix}$$

As in some other cases, it seems Ibn Sīnā may have been the first to make a formal move that we now recognise,
but his motivation was probably quite different from any we might have today.
In this case his thesis is about passage of information,
not about simplification of formal proofs.

Ibn Sīnā gives a number of examples of *luzūmī* (a,mt) sentences, but it is hard to see what significant feature they have in common besides being known to be true.

His extension to (e,mt) sentences is that these 'deny the *luzūm*'. But he himself points out two ways of reading this:

- They deny that the first clause entails the second.
- They deny the second clause, and this denial is entailed by the first clause.

My present impression is that Ibn Sīnā is casting around for a way of using this Peripatetic notion but has not yet succeeded in finding one.

How to say it in English?

Shehaby's translations for the *muttaṣil* sentences:

(a,mt) Always: when p, then q.
(e,mt) Never: when p, then q.
(i,mt) Sometimes: when p, then q.
(o,mt) Not always: when p, then q.

Close to Ibn Sīnā's Arabic, except for the colon which suggests a common constituent 'when p, then q' in all four. In fact this is wrong parsing, as we see from the fact that (e,mt) and (i,mt) both convert.

I suggest:

(a,mt) Whenever p then q.
(e,mt) It is never the case, when p, that q.
(i,mt) It can be the case, when p, that q.
(o,mt) It is not the case that whenever p then q.

A closer linguistic analysis suggests these readings for (e,mt) and (i,mt) may reflect how Ibn Sīnā's Arabic works too. Note that 'when p, that q' is not a constituent in either the (e,mt) sentence or the (i,mt).

For the *munfaṣil* sentences two cases are straightforward:

(a,mn) Always either p or q.
(o,mn) It is not the case that always either p or q.

(e,mn) and (i,mn) are harder. Problem: to match the Arabic while expressing the negation on the second clause.

I cautiously propose:

(e,mn) It is never the case that p while not that q.
(i,mn) It can be the case that p while not that q.

Note that (a,mn)(p,q) could be read as 'Always p, at least while not that q' in the non-strict reading and 'Always p, but only while not that q' in the strict. There is no case for using these strangled formulations in place of the usual 'Either … or'.

The words *muttaṣil* and *munfaṣil* themselves were certainly intended to describe the (a) forms, in the *muttaṣil* case with the existential augment at least implicit, and in the *munfaṣil* case with strictness implied.

Thus *muttaṣil*, from *wṣl* 'connect', reflects the fact that (a,mt) with augment implies that the conjunction of p and q (their logical 'meet') is sometimes true,
and *munfaṣil*, from *fṣl* 'separate', reflects the fact that (a,mn) with strictness expresses the logical symmetric difference of p and q.

Shehaby's translations 'connective' and 'separative' are good for the Arabic but mean nothing in logic.

I suggest

- 'meet-like' for *muttaṣil*
- 'difference-like' for *munfaṣil*

to match both Arabic and logic.

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